EFFECT OF HEAT CONVECTION ON THE DYNAMICS OF A LIQUID IN A ROTATING TANK

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The article investigates the dynamics of a viscous incompressible liquid in a rotating cylindrical tank in the presence of forces of heat convection.

The dynamics of a viscous liquid near a rotating disk was examined by Schlichting [1], and in a stationary tank under the effect of the rotating lid by Dorfman and Romanenko [2]. However, the operation of many hydropower installations containing a liquid involves rotation as well as thermal interaction with the environment. In this connection the following problem arises: to find the velocity and temperature field in the liquid in a rotating cylindrical tank whose walls are maintained at constant temperature below or above the initial temperature. It is assumed that the initial state is the state of rest with homogeneous distribution of the temperature T_0.

At an instant differing from zero, the tank containing a viscous incompressible liquid is made to rotate a constant angular velocity ω, and simultaneously the temperature of the tank walls is lowered or raised to the ambient temperature T_m. The rotation is effected around the vertical axis of symmetry – a cylindrical system of coordinates is used. We will also assume that the velocity vector $\vec{V}$ does not depend on the polar coordinate α. Here and henceforth all magnitudes are dimensionless, introduced in the standard way, and all linear dimensions are referred to the cylinder radius R.

The equations of motion written for vorticity and the circumferential component of the velocity, the equations of heat transfer and continuity have the form

$$\frac{1}{\mathrm{Pr}}\frac{\partial\varphi}{\partial\mathrm{Fo}}+\frac{\mathrm{Re}}{r}\left[\left(\frac{\varphi}{r}-\frac{\partial\varphi}{\partial r}\right)\frac{\partial\psi}{\partial z}+\frac{\partial\psi}{\partial r}\frac{\partial\varphi}{\partial z}\right]=\Delta\varphi+\frac{2}{r}\frac{\partial\varphi}{\partial r}-\frac{\varphi}{r^2}+\frac{2\mathrm{Re}\,U_\alpha}{r}\frac{\partial U_\alpha}{\partial z}+\mathrm{Gr}\frac{\partial\Theta}{\partial r};\qquad(1)$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial U_\alpha}{\partial\mathrm{Fo}}-\frac{\mathrm{Re}}{r}\left[\left(\frac{U_\alpha}{r}-\frac{\partial U_\alpha}{\partial r}\right)\frac{\partial\psi}{\partial z}-\frac{\partial\psi}{\partial r}\frac{\partial U_\alpha}{\partial z}\right]=\Delta U_\alpha+\frac{2}{r}\frac{\partial U_\alpha}{\partial r}-\frac{U_\alpha}{r^2};\qquad(2)$$

$$\frac{\partial\Theta}{\partial\mathrm{Fo}}-\mathrm{RePr}\left(\frac{\partial\psi}{\partial r}\frac{\partial\Theta}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial\Theta}{\partial r}\right)=\Delta\Theta-\frac{2}{r}\frac{\partial\Theta}{\partial r};\qquad(3)$$

$$-\varphi=\frac{1}{r}\Delta\psi.\qquad(4)$$

Here,

$$\Delta=\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2};\quad \varphi=\mathrm{curl}\,\vec{U};\quad U_r=-\frac{1}{r}\frac{\partial\psi}{\partial z};$$

$$U_z=\frac{1}{r}\frac{\partial\psi}{\partial r}.$$

System (1)-(4) was solved with the following boundary conditions:

For Fo = 0

$$U_\alpha=\varphi=\psi=0,\quad \Theta=1;\qquad(5)$$

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at the bottom and at the lid of the tank for z = 0 and z = l

$$\psi=\Theta=0,\quad U_\alpha=r,\quad \varphi=\frac{1}{r}\frac{\partial^2\psi}{\partial z^2}\,; \tag{6}$$

on the lateral wall for r = 1

$$\psi=\Theta=0,\quad U_\alpha=1,\quad \varphi=\frac{1}{r}\frac{\partial^2\psi}{\partial r^2}\,; \tag{7}$$

on the axis of symmetry for r = 0

$$U_\alpha=\psi=\varphi=0,\quad \frac{\partial\Theta}{\partial r}=0. \tag{8}$$

The problem formulated by (1)-(8) was solved by the finite-difference method by the monotonic procedure of [3] with uniform time Fo_h and coordinate ω_h grids (ω_h = 48 × 48). The boundary conditions for the vortex with velocity φ were approximated with the second order of accuracy in accordance with [4]. The sequence of solving the system (1)-(8) was selected as follows. First by longitudinal and transverse run, Eq. (2) was solved, then (3), and finally (1). The calculation process for the given temporary layer ended with the iteration of the Poisson equation (4) by the method of upper relaxation.

The effect of heat convection on the dynamics of a viscous liquid rotating in a tank was analyzed for a vessel with relative height l = 4 and constant Prandtl number Pr = 1. Figures 1-3 show the distribution of the isofunctions of flow and isotherms in the liquid region for different instants of time.

In Fig. 1 the results of the calculation were obtained for Re = 10^3, Pr = 1 and Gr = 50. Regardless of the small Grashof number, the symmetry of flow of the liquid relative to the center of the vertical height is infringed. The effect of heat convection is particularly noticeable at the initial and final stages of the process, which is perfectly natural.

Simultaneously with the beginning rotation of the tank, its walls assume the ambient temperature, i.e., a temperature lower than the initial one. Since the cooled layers near the tank walls are denser, the direction of the force of convective motion in the upper half of the vessel coincides with the centrifugal forces acting on the liquid. As a result, symmetry is infringed: the upper vortex assumes a dominant position (Fig. 1, Fo = 0.01).

As the liquid is ever-more carried away by the tank walls, the nature of its motion changes: the so-called convective instability of flow (Fo = 0.03; Fo = 0.05) arises. And, once the liquid in the tank rotates fully, the forces of convective motion again predominate (Fo = 0.07; Fo = 0.09). It should be pointed out that the velocity of the liquid with the selected parameters has a fairly weak influence on the temperature field of the medium although there is some tendency toward displacement of the thermal center toward the upper part of the tank.

With increasing Grashof number Gr = 100 (the other parameters remaining unchanged), the effect of heat convection becomes ever-more noticeable (Fig. 2). The effect of the convective motion of the temperature field becomes more substantial. The time interval within which the velocity field is rearranged is considerably reduced. Further intensification of heat convection (Gr = 1000 or more) with unchanged angular velocity of the rotation of the cylinder is an ever-increasing obstacle to the origin of distortions in the liquid medium due to centrifugal forces. In particular, with Reynolds and Grashof numbers equal to Re = 500 and Gr = 10^4, respectively, the motion of the liquid occurs only under the effect of the inhomogeneity of the temperature field. In this case, one closed vortex forms in the liquid medium: near the lateral walls of the tank, the streams are directed downward, and along the axis of symmetry — upward. Along the coordinate α, the liquid moves like an absolutely solid body.

With Reynolds numbers increasing to Re = 2000 while the Grashof numbers, as before, remain small (Gr $\leq$ 500), again a time interval can be observed characterized by the rearrangement of the velocity field under the effect of centrifugal and gravitational forces. The higher Grashof numbers are due to the motion of the liquid in the radial cross section of the rotating cylinder under the effect of heat convection only.

With negative Grashof numbers corresponding to the heating of the liquid medium by the tank walls, the flow shows a number of specific features. In this case the liquid, under

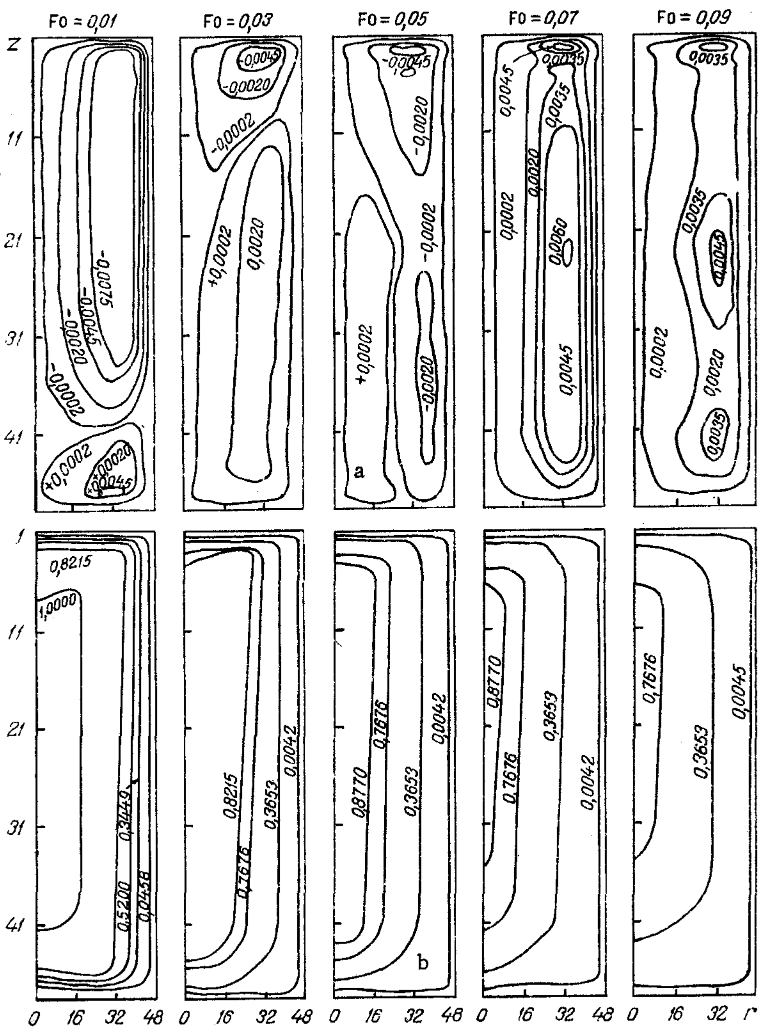

Fig. 1

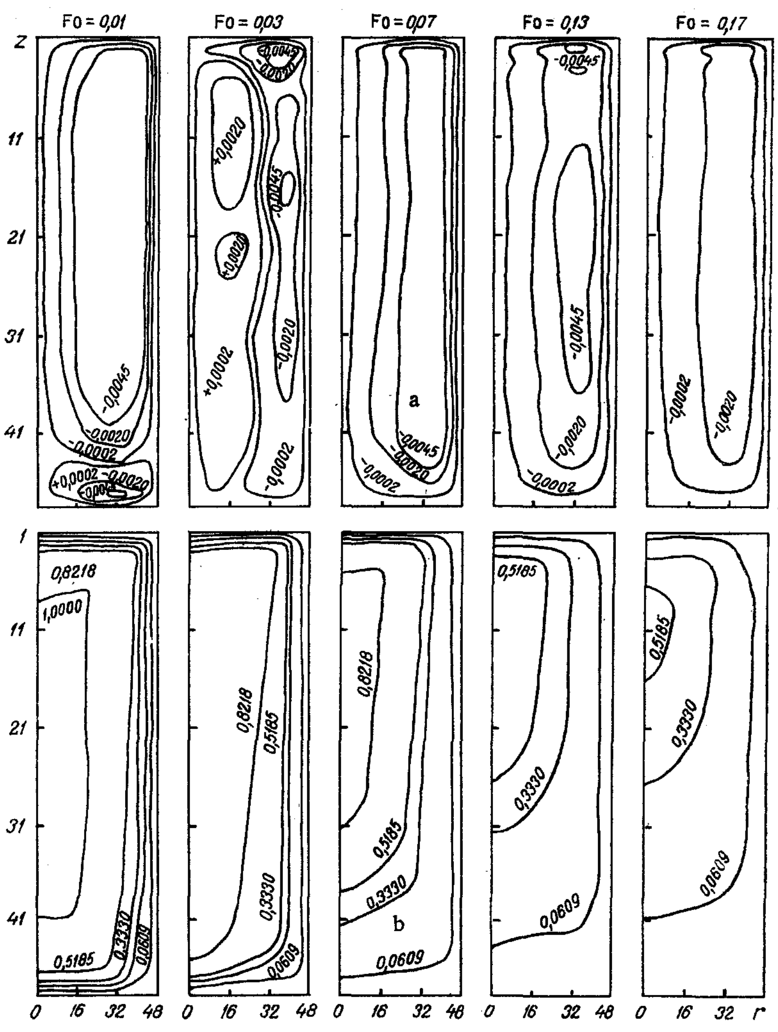

Fig. 2

Fig. 1. Distribution of the stream function (a) and isotherms (b) in the liquid of a rotating tank: Gr = 50; Re = 1000; l = 4; Pr = 1.

Fig. 2. Distribution of the stream function (a) and isotherms (b) in the liquid of a rotating tank: Gr = 100; Re = 1000; l = 4; Pr = 1.

Fig. 3. Distribution of the stream function (a) and isotherms (b) in the liquid of a rotating tank: Gr = 1000; Re = 2000; l = 4; Pr = 1.

the effect of the inhomogeneity of the temperature field, moves upward near the walls, and downward along the axis of symmetry. The counteraction of the centrifugal and gravitational forces has the effect that over the height, the regions of flow are of a cellular nature. Figure 3 traces well the consecutive formation of such a structure. Also noticeable is the effect of the velocity field on the temperature field of the liquid region. It can be seen from Fig. 3b that the thermal center is concentrated at the given part of the tank.

Thus, the interaction between centrifugal and gravitational forces in the liquid of a rotating tank fully determines the structure of its flow. In the region of laminar flow, small Grashof numbers affect the initial and final stages, and with Gr > 1000, the effect of the centrifugal forces becomes practically imperceptible.

NOTATION

Fo = at/R^2, Fourier number; a, thermal diffusivity; t, time; Re = $\omega R^2/\nu$, Reynolds number; ν, kinematic viscosity; Gr = $\beta g(T_0 - T_m)R/\omega/\nu$, Grashof number; g, acceleration of gravity; β, coefficient of thermal expansion.

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APPROXIMATE SOLUTION OF A PROBLEM OF UNSTEADY CONVECTIVE HEAT EXCHANGE IN LONGITUDINALLY STREAMLINED BUNDLES OF RODS

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Unsteady heat exchange is investigated in a longitudinal bundle of rods with a square arrangement during laminar, hydrodynamically stabilized flow of a viscous, incompressible fluid.

In calculations of heat-exchanging systems it is of interest to investigate convective heat exchange in longitudinal flow past bundles of rods. The steady heat exchange in bundles has been studied in [1-3], and elsewhere.

We consider the problem of unsteady heat exchange in a bundle of semi-infinite rods with a square arrangement, with laminar flow of a viscous incompressible fluid. Neglecting the heat of friction and the axial thermal conductivity, assuming the fluid flow to be steady and hydrodynamically stabilized, and the thermophysical properties of the fluid to be constant, we write the energy equation for determination of the temperature field

$$\frac{\partial T}{\partial t}+\omega_z(r,\ \varphi)\frac{\partial T}{\partial z}=a\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}\right)$$
$$(t>0;\quad z>0;\quad r,\ \varphi\in\Omega),$$

where $T(r, \varphi, z, t)$ is the unknown temperature field; t, time; the z axis is collinear with the direction of the fluid flow; r and φ, cylindrical coordinates; the region $\Omega:\{0 < \varphi < \pi/4,\ r_0 < r < b/\cos\varphi\}$ represents a translational element of the bundle, by consideration of which we can limit ourselves by virtue of the symmetry of the rod system (Fig. 1); $\omega_z(r,\varphi)=\frac{\Delta p b^2}{l\mu}\left\{\frac{2}{\pi}\ln\frac{r}{r_0}-\frac{1}{4}\left[\left(\frac{r}{b}\right)^2-\left(\frac{r_0}{b}\right)^2\right]+\sum_{j=1}^{m}\frac{\varepsilon_j}{4j}\left(\frac{r}{b}\right)^{4j}\left[1-\left(\frac{r_0}{r}\right)^{8j}\right]\cos 4j\varphi\right\}$ velocity profile of the fluid flow in a translational element [4].

We assume that at the entrance to the bundle the fluid temperature equals $T_e(r, \varphi, t)$, and at the initial moment ot time $T_i(r, \varphi, z)$. Beginning with $t = 0^+$ the temperature of the external surface of the rod assumes the value $T_S(z, t)$. Hence, for the region under consideration the initial and boundary conditions take the form

$$T(r,\ \varphi,\ z,\ 0)=T_i(r,\ \varphi,\ z);\quad T(r,\ \varphi,\ 0,\ t)=T_e(r,\ \varphi,\ t);$$
$$T(r_0,\ \varphi,\ z,\ t)=T_S(z,\ t);\quad \left.\frac{\partial T}{\partial \nu}\right|_{\Gamma_1\cup\Gamma_2\cup\Gamma_3}=0,$$

where $\Gamma_1:\{r_0 < r\leqslant b,\ \varphi = 0\}$; $\Gamma_2:\{r = b/\cos\varphi,\ 0\leqslant\varphi\leqslant\pi/4\}$; $\Gamma_3:\{r_0 < r\leqslant\sqrt{2}b,\ \varphi=\pi/4\}$; ν is the interior normal to the boundary Ω.

The boundary-value problem in dimensionless form will be

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